

5D gravity and the discrepant G measurements

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Abstract

It is shown that 5D Kaluza-Klein theory stabilized by an external bulk scalar field may solve the discrepant laboratory G measurements. This is achieved by an effective coupling between gravitation and the geomagnetic field. Experimental considerations are also addressed.

1 Introduction

Although the methods and techniques have been greatly improved since the late nineteenth century, the precision on the measurement of the gravitational constant, G , is still the less accurate in comparison with the other fundamental constants of nature [1]. Moreover, given the relative uncertainties of most of the individual experiments (reaching about 10^{-4} for the most precise measurements), they show an incompatibility which leads to an overall precision of only about 1 part in 10^3 . Thus the current status of the G terrestrial measurements (see [2]) implies either an unknown source of errors (not taken into account in the published uncertainties), or some new physics [3]. In the latter spirit, many theories which include extradimensions have been proposed as candidates for the unification of physics. As such, they involve a coupling between gravitation and electromagnetism (GE coupling), as well as with other gauge fields present.

Here we show that the discrepancy between the present results of the G-measurements may be understood as a consequence of the GE coupling. Also, this theory predicts a variation of the effective fine structure "constant" α with the gravitational field, and thus with the cosmological time (see [4]).

2 Theoretical background

An argument initially from Landau and Lifshitz [5] may be applied to the pure Kaluza-Klein (KK) action (see [6]): the negative sign of the kinetic term of the five dimensional (5D) KK internal scalar field, Φ , leads to inescapable instability. The question to know which of the two conformally related frames (Einstein-Pauli frame or Jordan-Fierz frame) is the best remains debated in the literature, each frame having its own advantage. The negative kinetic energy density for the Φ -field, and thus the instability, occur in both frames. In the following, we perform calculations in the Jordan-Fierz frame, where, as we show, the discrepant laboratory measurements of G find a natural explanation. Also, it is true that the 5D KK theory may yield a zero kinetic term (and thus a zero kinetic energy density, which would also be unusual in 4D), but this occurs only when the electromagnetic (EM) field is identically zero everywhere. This may be relevant for some cosmological solutions, but not for our discussion. On the contrary, we argue here that the Φ -field is tightly related to the EM field via the link between the compactified space of the fifth dimension and the U(1) gauge group.

Since stabilization may be obtained if an external field is present, we assume here a version, $\text{KK}\psi$, which includes an external bulk scalar field minimally coupled to gravity (like the radion in the brane world scenario). After dimensional reduction ($\alpha = 0, 1, 2, 3$), this bulk field reduces to a four dimensional scalar field $\psi = \psi(x^\alpha)$ and, in the Jordan-Fierz frame, the low energy effective action takes the form (up

to a total divergence)

$$S = - \int \sqrt{-g} \left[\frac{c^4}{16\pi} \frac{\Phi}{G} R + \frac{1}{4} \varepsilon_0 \Phi^3 F_{\alpha\beta} F^{\alpha\beta} + \frac{c^4}{4\pi G} \frac{\partial_\alpha \Phi \partial^\alpha \Phi}{\Phi} \right] d^4x \\ + \int \sqrt{-g} \Phi \left[\frac{1}{2} \partial_\alpha \psi \partial^\alpha \psi - U - J\psi \right] d^4x, \quad (1)$$

where A^α is the potential 4-vector of the EM field, $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ the EM field strength tensor, U the self-interaction potential of ψ of the symmetry breaking type and J its source term.

The source term of the ψ -field, J , includes the contributions of the ordinary matter, of the EM field and of the internal scalar field Φ . For each, the coupling is defined by a function (temperature dependent, as for the potential U) $f_X = f_X(\psi, \Phi)$, where the subscript X stands for "matter", "EM" and " Φ ". In addition, the necessity to recover the Einstein-Maxwell equations in the weak fields limit, implies the following conditions: $U(v) = 0$ and $f_{EM}(v, 1) = f_{matter}(v, 1) = 0$, where v denotes the vacuum expectation value (VEV) of the ψ -field.

The contributions of matter and Φ are proportional to the traces of their respective energy-momentum tensors. A contribution of the form $\varepsilon_0 f_{EM} F_{\alpha\beta} F^{\alpha\beta}$ accounts for the coupling with the EM field. Besides, though the fit to the data involves $\frac{\partial f_{EM}}{\partial \Phi}(v, 1) v \gg 4\pi G/c^4$, we may infer that $\frac{\partial f_{EM}}{\partial \Phi}(v, 1) v$ is negligibly small at very high temperature (e.g., like in the core of the Sun or at the big bang nucleosynthesis) and even vanishes beyond the critical temperature (say $T_c = 6000$ K) of the potential $U = U(\psi, T)$ as one gets $v = 0$ in that case.

Following Lichnerowicz [7], let us interpret the quantity

$$G_{eff} = \frac{G}{\Phi} \quad (2)$$

of the Einstein-Hilbert term, and the factor $\varepsilon_{0eff} = \varepsilon_0 [\Phi^3 + 4f_{EM}(\psi, \Phi)]$ of the Maxwell term respectively as the effective gravitational "constant" and the effective vacuum dielectric permittivity. The effective vacuum magnetic permeability reads

$\mu_{0eff} = \mu_0 [\Phi^3 + 4f_{EM}(\psi, \Phi)]^{-1}$, so that the velocity of light in vacuum remains a true universal constant. Both terms depend on the local (for local physics) or global (at cosmological scale) value of the KK scalar field Φ , assumed to be positive defined.

The least action principle applied to the action (1) yields the generalized Einstein-Maxwell equations and the scalar fields equations

$$\nabla_\nu \nabla^\nu \psi = -J - \frac{\partial J}{\partial \psi} \psi - \frac{\partial U}{\partial \psi} \quad (3)$$

and

$$\nabla_\nu \nabla^\nu \Phi = -\frac{4\pi G}{c^4} \varepsilon_0 F_{\alpha\beta} F^{\alpha\beta} \Phi^3 + U \Phi + J \psi \Phi + \frac{\partial J}{\partial \Phi} \Phi^2 \psi - \frac{1}{2} (\partial_\alpha \psi \partial^\alpha \psi) \Phi. \quad (4)$$

3 Solutions in presence of a dipolar magnetic field

Let us study the *spatial* variation of Φ in the weak fields conditions out of the fields' source, but in presence of a static dipolar magnetic field, $\vec{B} = \vec{\nabla} V(r, \varphi, \theta)$. We denote r , φ and θ respectively the radius from the centre, the azimuth angle and the colatitude. Thus, writing $\Phi = \Phi(r, \varphi, \theta)$, and taking into account that $\frac{\partial U}{\partial \psi}(v) = 0$ (definition of the VEV), equation (4) simplifies after linearization as

$$\Delta \Phi = -\frac{2}{\mu_0} \left[\frac{\partial f_{EM}}{\partial \Phi}(v, 1) v + \frac{4\pi G}{c^4} \right] (\vec{\nabla} V)^2. \quad (5)$$

Since $\Delta V = \text{div } \vec{B} = 0$ and $(\vec{\nabla} V)^2 = \frac{1}{2} \Delta(V^2) - V \Delta V$ identically, the solution of equation (5) above reads merely

$$\Phi = 1 - \frac{1}{\mu_0} \left[\frac{\partial f_{EM}}{\partial \Phi}(v, 1) v + \frac{4\pi G}{c^4} \right] V^2. \quad (6)$$

For our purpose, it is sufficient to limit the expansion of the scalar potential, V , to the terms of the Legendre function of degree one ($n = 1$) and order one ($m = 1$). Hence $V = (a^3/r^2) [g_1^0 \cos \theta + g_1^1 \sin \theta \cos \varphi + h_1^1 \sin \theta \sin \varphi]$,

where g_1^0 , g_1^1 and h_1^1 are the relevant Gauss coefficients, a is the Earth's radius and $M = \frac{4\pi}{\mu_0} a^3 \sqrt{(g_1^0)^2 + (g_1^1)^2 + (h_1^1)^2}$ denotes its magnetic moment. Setting $\cos \varphi_1 = -g_1^1 / \sqrt{(g_1^1)^2 + (h_1^1)^2}$, $\sin \varphi_1 = h_1^1 / \sqrt{(g_1^1)^2 + (h_1^1)^2}$ and $\tan \lambda = g_1^0 / \sqrt{(g_1^1)^2 + (h_1^1)^2}$, the solution of equation (5) then reads (and similarly for ψ by making the substitution $\frac{\partial f_{EM}}{\partial \Phi} \rightarrow -\frac{\partial f_{EM}}{\partial \psi}$)

$$\Phi = 1 - \frac{1}{\mu_0} \frac{\partial f_{EM}}{\partial \Phi} (v, 1) v \left(\frac{\mu_0 M}{4\pi r^2} \right)^2 x(\theta, \varphi), \quad (7)$$

where we have set

$$x(\theta, \varphi) = \cos^2 \theta + \cot^2 \lambda \sin^2 \theta \cos^2 (\varphi + \varphi_1) - \cot \lambda \sin 2\theta \cos (\varphi + \varphi_1). \quad (8)$$

Thence, one derives the expression of $G_{eff}(r, \theta, \varphi)$ by inserting the solution (7) above in relation (2).

It is worth noticing that the magnetic potential, V , scales as $B r$, where B is the magnitude of the geomagnetic field at radius r . Indeed, because of this scaling effect, the small spatial variations of the geomagnetic field will influence significantly the laboratory measurements of G whereas the large local magnetic fields present in the laboratory (e. g., the magnetic suspension used to support the balance beam, the magnetic damper, etc...) will not. A rough estimate shows that, even using a 30 Tesla superconducting magnet, one still needs to gain at least one order of magnitude with the most precise G measuring apparatus available yet. Hence, our prediction is consistent with the earlier conclusion of Lloyd [11].

4 Comparison with laboratory measurements

There are presently almost 45 results of G measurements published since 1942 [12]. Because of the too numerous uncontrolled systematic biases, the mine measurements are excluded from the present study (including them will not change our fit because of their lack of precision, typically less than 1%). Also, the more discordant laboratory measurement (high PTB value [8]) is excluded, since it may suffer from a

systematic effect. The " official " values are presently $G = 6.67259 \pm 0.00085 \cdot 10^{-11}$ (CODATA 86, [9]) and $G = 6.670 \pm 0.010 \cdot 10^{-11}$ (CODATA 2000, [10]) in MKS unit. In the following, all the measurements are weighted equally in the fit. Fitting the 44 data with these values gives respectively $\chi^2_\nu = 11.128$ and $\chi^2_\nu = 62.498$ ($\chi^2_\nu = \chi^2$ per degrees of freedom). If we forget the official values and try a best fit, assuming an arbitrary constant value of G , we obtain $G = 6.6741 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ with $\chi^2_\nu = 2.255$. The fit to the same sample of 44 measurements (figure 1), on account of the GE coupling, yields (in MKS units) with $\chi^2_\nu = 1.669$:

$$\frac{1}{10^{11} G_{eff}} = (0.149929 \pm 0.000017) - (0.0001509 \pm 0.0000252) x(L, l). \quad (9)$$

From the above fit, one derives both estimates of the true gravitational constant

$$G = (6.6696 \pm 0.0008) \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}, \quad (10)$$

and the coupling parameter

$$\frac{\partial f_{EM}}{\partial \Phi}(v, 1) v = (5.44 \pm 0.66) \cdot 10^{-6} \text{ fm TeV}^{-1}. \quad (11)$$

The latter quantity, expressed in the canonical form $\frac{\partial f_{EM}}{\partial \Phi}(v, 1) v = \hbar c M_5^{-2}$, yields a 5D Planck scale $M_5 \simeq 5.9 \text{ TeV}$ of the order of the value that is invoked in the literature to solve the hierarchy problem.

<i>Location [reference]</i>	<i>Latitude (°)</i>	<i>Longitude (°)</i>	<i>G_{lab} (10⁻¹¹ m³ kg⁻¹ s⁻²)</i>
Lower Hutt (MSL) [16, 15]	-41.2	174.9	6.6742 ± 0.0007 6.6746 ± 0.0010
Wuhan (HUST) [17]	30.6	106.88	6.6699 ± 0.0007
Los Alamos [18]	35.88	-106.38	6.6740 ± 0.0007
Gaithersburg (NBS) [19, 20]	38.9	-77.02	6.6726 ± 0.0005 6.6720 ± 0.0041
Boulder (JILA) [21]	40	-105.27	6.6873 ± 0.0094
Gigerwald lake [22, 23]	46.917	9.4	6.669 ± 0.005 (at 112 m) 6.678 ± 0.007 (at 88 m) 6.6700 ± 0.0054
Zurich [24, 25]	47.4	8.53	6.6754 ± 0.0005 ± 0.0015 6.6749 ± 0.0014
Budapest [26]	47.5	19.07	6.670 ± 0.008
Seattle [14]	47.63	- 122.33	6.674215 ± 0.000092
Sevres (BIPM) [27, 28]	48.8	2.13	6.67559 ± 0.00027 6.683 ± 0.011
Fribourg [29]	46.8	7.15	6.6704 ± 0.0048 (Oct. 84) 6.6735 ± 0.0068 (Nov. 84) 6.6740 ± 0.0053 (Dec. 84) 6.6722 ± 0.0051 (Feb. 85)
Magny-les-Hameaux [30]	49	2	6.673 ± 0.003
Wuppertal [31]	51.27	7.15	6.6735 ± 0.0011 ± 0.0026
Braunschweig (PTB) [8, 32]	52.28	10.53	6.71540 ± 0.00056 6.667 ± 0.005
Moscow [33, 34]	55.1	38.85	6.6729 ± 0.0005 6.6745 ± 0.0008
Dye 3, Greenland [35]	65.19	-43.82	6.6726 ± 0.0027
Lake Brasimone [36]	43.75	11.58	6.688 ± 0.011

Table 1 : Results of the most precise laboratory measurements of G published during the last sixty years and location of the laboratories.

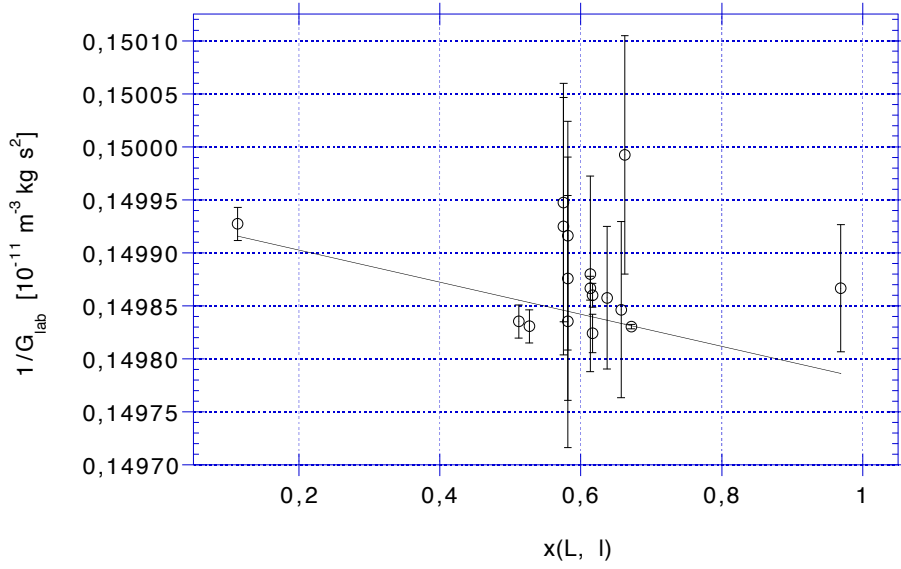


Figure 1: Laboratory measurements with relative uncertainty $\frac{\delta G_{lab}}{G_{lab}} < 10^{-3}$ and measuring time $\Delta t < 200$ s (sample S1, 17 points [16], [17] - [20], [23], [14], [29] - [31], [32] - [35]). The line indicates the best fit G_{lab} versus the mixed variable x ($\chi^2_\nu = 1.327$). Assuming a constant G would yield a bad fit to the data ($\chi^2_\nu = 3.607$), mostly because of the HUST value.

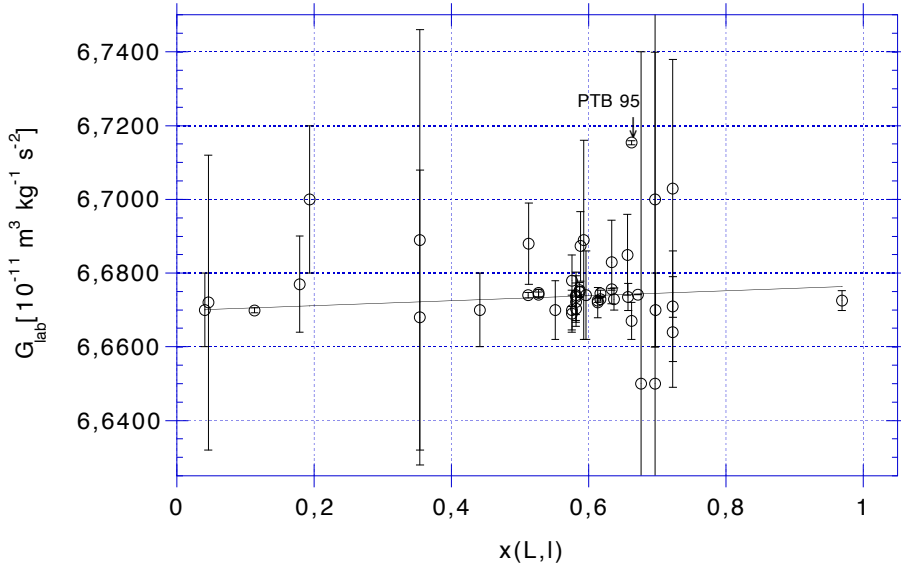


Figure 2: G_{lab} versus x (whole sample plus the PTB 95 value, 45 points [15] - [37]).

Sample	H0	H1
S1 17 points (Fig.1)	$\chi_\nu^2 = 3.607$ (best fit) $\chi_\nu^2 = 21.523$ (mean of CODATA 86) $\chi_\nu^2 = 141.46$ (mean of CODATA 2000)	$\chi_\nu^2 = 1.327$
Whole [16] - [37] 44 points (Fig.2)	$\chi_\nu^2 = 2.255$ (best fit) $\chi_\nu^2 = 11.128$ (mean of CODATA 86) $\chi_\nu^2 = 62.498$ (mean of CODATA 2000)	$\chi_\nu^2 = 1.669$

Table 2 : Reduced χ^2 for the two different hypothesis H0 (Hypothesis of a constant G) and H1 (Hypothesis of an effective G), and different samples S1 and whole (except the high PTB value [8], see text).

Considering the whole sample, we check the relevance of our result under H1 compared to the best value of G under H0, by applying the F test (Fisher law). This yields $F_\chi = \frac{\Delta\chi^2}{\chi_\nu^2} = 16.09$, which indicates that, independently of the number of parameters (two instead of one), our fit is better with a significance level greater than 99.9% [13]. Let us emphasize that the most precise value of G today [14] contributes to χ_ν^2 to less than 0.0006 in the above fit. Likewise, the last published value of G [38] would contribute to less than 0.02. This suggests that the agreement may be better than purely indicated by the χ^2 values. Moreover, if one substitutes $G = (6.6731 \pm 0.0002) 10^{-11} m^3 kg^{-1} s^{-2}$ [39] announced by the MSL team in June 2002 (CPM, Ottawa, Canada) for the prior $G = (6.6742 \pm 0.0007) 10^{-11} m^3 kg^{-1} s^{-2}$ published in 1999 [16], one would obtain with the sample S1 :

$$\text{H0 : } \chi_\nu^2 = 5.168$$

$$\text{H1 : } \chi_\nu^2 = 1.162$$

and with the whole sample of 44 measurements :

$$\text{H0 : } \chi_\nu^2 = 2.836$$

$$\text{H1 : } \chi_\nu^2 = 1.488$$

5 Discussion and Conclusion

It is worth noticing that the scalar fields under considerations identify neither to the dilaton nor to the inflaton of higher dimensional theories, without further assumptions. In particular, the computation of both scalar fields, as given by equations (3) and (4), involves only (see the right hand sides) quantities related to fields sources, and not to the test bodies. Hence, the effective G_{eff} given by relation (2) does not depend on the composition of the test bodies. Besides, let us emphasize that the equation of motion of a neutral point-like particle in the genuine KK theory reduces to the 4D geodesic equation after dimensional reduction [40], although the KK scalar field is coupled to the Maxwell invariant $F_{\alpha\beta} F^{\alpha\beta}$. In a forthcoming paper [41], we address the effect of the varying effective coupling constants on the masses of composite particles. On account of the Higgs mechanism of quarks and leptons masses generation, by promoting the Yukawa coupling constants to effective parameters (on an equal footing with G or α) that depend on both scalar fields ψ and Φ , we prove that the KK ψ model is actually consistent with the current experimental bounds on the violation of the equivalence principle. Hence, we conclude that present laboratory experiments may not measure a true constant of gravitation. Instead, in addition to all other possible biases (e. g., anelasticity in the wire of torsion pendulum as pointed out by Kuroda [42], and which since has been generically taken into account in the experiments), they may be pointing out an effective one depending on the geomagnetic field at the laboratory position.

References

- [1] J. Luo and Z. K. Hu, (2000), *Class. Quantum Grav.* **17**, 2351.
- [2] G. T. Gillies, *Rep. Prog. Phys.* **60**, (1997), 151, and references therein.

- [3] V. N. Melnikov, gr-qc/9903110.
- [4] J.-P. Mbelek and M. Lachièze-Rey, (2003), *A & A* **397**, 803.
- [5] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Addison-Wesley Publishing Company Inc., Reading, Massachusetts, 1959), pp. 289-293.
- [6] J.-P. Mbelek and M. Lachièze-Rey, gr-qc/0012086, and references therein.
- [7] A. Lichnerowicz, *Théories relativistes de la gravitation et de l'électromagnétisme*, (Masson et C^{ie}, Paris, France, 1955), pp. 201-206.
- [8] W. Michaelis, H. Haars and R. Augustin, (1995/96), *metrologia* **32**, 267.
- [9] E. R. Cohen and B. N. Taylor, (1987), *Rev. Mod. Phys.* **59**, 1121.
- [10] P. J. Mohr and B. N. Taylor, (2000), *Rev. Mod. Phys.* **72**, 351.
- [11] M. G. Lloyd, (1909), *Terrestrial Magnetism and Atmospheric Electricity* **14**, 67.
- [12] J.-P. Mbelek and M. Lachièze-Rey, (2002), *Grav. Cosmol.* **8**, 331.
- [13] P. R. Bevington, *Data reduction and error analysis for the physical sciences*, (McGraw-Hill Book Company, New York, 1969), pp. 200-203 and pp. 317-323.
- [14] J. H. Gundlach and S. M. Merkowitz, (2000), *Phys. Rev. Lett.* **85**, 2869.
- [15] M. P. Fitzgerald and T. R. Armstrong, 1995, *IIIE Trans. Instrum. Meas.* **44**, 494.
- [16] M. P. Fitzgerald and T. R. Armstrong, (1999), *Meas. Sci. Technol.* **10**, 439.
- [17] J. Luo *et al.*, (1998), *Phys. Rev.* **D59**, 042001.
- [18] C. H. Bagley and G. G. Luther, (1997), *Phys. Rev. Lett.* **78**, 3047.

- [19] G. G. Luther and W. Towler, (1982), Phys. Rev. Lett. **48**, 121.
- [20] P. R. Heyl and P. Chrzanowski, (1942), J. Res. Nat. Bur. Standards **29**, 1.
- [21] J. P. Schwarz *et al.*, (1998), Science **282**, 2230.
- [22] B. Hubler, A. Cornaz and W. Kündig, (1995), Phys. Rev. **D51**, 4005.
- [23] A. Cornaz, B. Hubler and W. Kündig, (1994), Phys. Rev. Lett. **72**, 1152.
- [24] J. Schurr, F. Nolting and W. Kündig, (1998), Phys. Rev. Lett. **80**, 1142.
- [25] F. Nolting, J. Schurr, S. Schlamming and W. Kündig, (1999), Meas. Sci. Technol. **10**, 487.
- [26] J. Renner, in *Determination of Gravity Constant and Measurement of Certain Fine Gravity Effects*, Y. D. Boulanger and M. U. Sagitov (Eds.), (National Aeronautics and Space Administration, Washington, 1974), pp. 26-31.
- [27] T. J. Quinn, C. C. Speake, S. J. Richman, R. S. Davis and A. Picard, (2001), Phys. Rev. Lett. **87**, 111101.
- [28] S. J. Richman, T. J. Quinn, C. C. Speake and R. S. Davis, (1999), Meas. Sci. Technol. **10**, 460.
- [29] J. -Cl. Dousse and Ch. Rhône, (1987), Am. J. Phys. **55**, 706.
- [30] L. Facy and C. Pontikis, (1971), Comptes Rendus des Scéances de l'Académie des Sciences de Paris **272**, Série B, 1397.
- [31] U. Kleinevoss, H. Meyer, A. Schumacher and S. Hartmann, (1999), Meas. Sci. Technol. **10**, 492.
- [32] H. de Boer, H. Haars and W. Michaelis, (1987), metrologia **24**, 171.

- [33] O. V. Karagioz, V. P. Izmaylov and G. T. Gillies, (1998), *Grav. Cosmol.* **4**, 239.
- [34] M. U. Sagitov *et al.*, (1979), *Dok. Acad. Nauk SSSR* **245**, 567.
- [35] M. A. Zumberge *et al.*, (1990), *J. Geophys. Res.* **95**, 15483.
- [36] P. Baldi *et al.*, (2001), *Phys. Rev.* **D64**, 082001.
- [37] G. Müller *et al.*, (1990), *Geophys. J. Int.* **101**, 329 ; M. A. Zumberge *et al.*, (1991), *Phys. Rev. Lett.* **67**, 3051 ; M. Oldham, F. J. Lowes and R. J. Edge, (1993), *Geophys. J. Int.* **113**, 83 ; J. K. Hoskins *et al.*, (1985), *Phys. Rev.* **D32**, 3084 ; Y. T. Chen *et al.*, (1984), *Proc. R. Soc. Lond.* **A394**, 47 ; R. D. Rose *et al.*, (1969), *Phys. Rev. Lett.* **23**, 655 ; G. I. Moore *et al.*, (1988), *Phys. Rev.* **D38**, 1023 ; F. D. Stacey *et al.*, (1987), *Rev. Mod. Phys.* **59**, 157 ; X. Yang *et al.*, (1991), *Chinese Phys. Lett.*, **8**, 329 ; M. S. Saulnier and D. Frisch, (1989), *Am. J. Phys.*, **57**, 417 ; G. J. Tuck *et al.*, in *The Fifth Marcel Grossmann Meeting*, D. G. Blair and M. J. Buckingham (Eds.), (World Scientific, Singapore, 1989), pp. 1605-1612 ; K. Kuroda and H. Hirakawa, (1985), *Phys. Rev.* **D32**, 342.
- [38] St. Schlamminger, E. Holzschuh and W. Kündig, (2002), *Phys. Rev. Lett.* **89**, 161102.
- [39] R. Newman, in J. Pullin (Ed.), gr-qc/0303027.
- [40] P. S. Wesson and J. Ponce de Leon, (1995), *A & A* **294**, 1 ; J. Ponce de Leon, (2002), *Grav. Cosmol.* **8**, 272.
- [41] J.-P. Mbelek and M. Lachièze-Rey, to appear.
- [42] K. Kuroda, (1995), *Phys. Rev. Lett.* **75**, 2796.